

say, a few seconds.¹¹ This is enough to exclude the possible assignment $J=6$ for the level e , which would otherwise be acceptable. The internal conversion of the quadrupole gamma-ray by electron or pair emission is very improbable. No positrons have yet been observed in spite of fairly careful search. Further confirmation of

¹¹ See reference b of Table I.

the scheme seems possible but not easy, either by looking still more closely for the predicted positrons or by improving the accuracy of the K -capture and gamma-ray yield measurements considerable.

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On Fermi's Theory of the Origin of Cosmic Radiation

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Starting from Fermi's theory of the origin of cosmic radiation, an explanation of the observed shape of the energy spectrum of the proton component is suggested. On this basis there seems to be some indication that the majority of the protons originate from the nucleus of our galaxy. Assuming that the nucleus of the galaxy is a cluster of stars, the origin of the heavier nuclear components and protons of cosmic radiation can be explained on the same basis providing that an ordinary star can eject high energy particles (up to one Bev per nucleon). The observation of the effect of the sun on the heavier nuclear components may give a critical test of the theory.

The trapping magnetic field, if any, of the galaxy has a negligible effect on the energy spectra of cosmic radiation observed on the earth. The age of the galaxy is estimated to be greater than two billion years.

I. INTRODUCTION

RECENT measurements of the proton component of the cosmic radiation^{1,2} show that the differential energy spectrum indicates a maximum at the low energy end with a gradually increasing slope toward high energies. In this paper, which is based on Fermi's theory of the origin of cosmic radiation,³ an explanation of this effect is suggested and a hypothetical picture describing the origin of cosmic radiation is then deduced.

Fermi's theory is based on the assumed existence of wandering magnetic clouds in interstellar space. According to this theory a charged particle will gain energy by a "head-on collision" with a magnetic cloud and will lose energy by an "over-taking collision," but since the chance of a collision of the first type is larger than one of the second type, there is an average energy gain per collision. This is given by $\delta w = B^2 w$, where w is the energy of the particle (including rest energy) before the collision, and B is the velocity of the wandering cloud in units of the velocity of light. If the initial energy of the particle exceeds a certain threshold value so that the loss of energy in passing through the interstellar medium between collisions can be compensated by the gain, the particle will be accelerated and will become a cosmic-ray particle.

The wandering magnetic clouds are presumably eddies in interstellar space. They are stirred up by the rotation of the galaxy. Recent reports on the polarization of starlight when passing through interstellar clouds

seem to support the existence of the magnetic nature of these clouds.⁴⁻⁶ The existence of the wandering magnetic clouds indicates that Fermi's mechanism must account for at least a part of the cosmic radiation. We shall start with Fermi's mechanism as a fundamental basis and investigate the spatial distribution of sources of protons which will produce the observed spectrum of the primary proton component.

Assume that protons originate from some source at a distance r from the earth and that some of them reach the earth after many collisions. Applying the principle of random flight, we are able to deduce the energy distribution of these protons as a function of r . Thus, the energy spectrum of the proton component is determined by the spatial distribution of the proton sources. Several types of spatial distribution were investigated, and it was found that the observed energy spectrum seems to indicate that the majority of cosmic-ray particles originate from the nucleus of our galaxy.

The nucleus of our galaxy is presumably a cluster of stars. If stars are able to eject high energy particles (up to one Bev per nucleon), as proposed by different authors,^{7,8} then our finding might mean that cosmic-ray particles originate from stars and are then accelerated in interstellar space according to Fermi's mechanism. However, if normal stars are unable to eject high energy particles, we are forced to assume that a series of some

⁴ J. S. Hall, *Science* **109**, 166 (1949).

⁵ W. A. Hiltner, *Science* **109**, 165 (1949).

⁶ L. Spitzer and J. W. Tukey, *Science* **109**, 461 (1949).

⁷ F. Hoyle, *Monthly Notices Roy. Astron. Soc.* **106**, 384 (1947).

⁸ K. Kiepenheuer, *Phys. Rev.* **78**, 809 (1950).

¹ Winckler, Stix, Dwight, and Sabin, *Phys. Rev.* **79**, 656 (1950).

² J. A. Van Allen and S. F. Singer, *Phys. Rev.* **78**, 819 (1950).

³ E. Fermi, *Phys. Rev.* **75**, 1169 (1949).

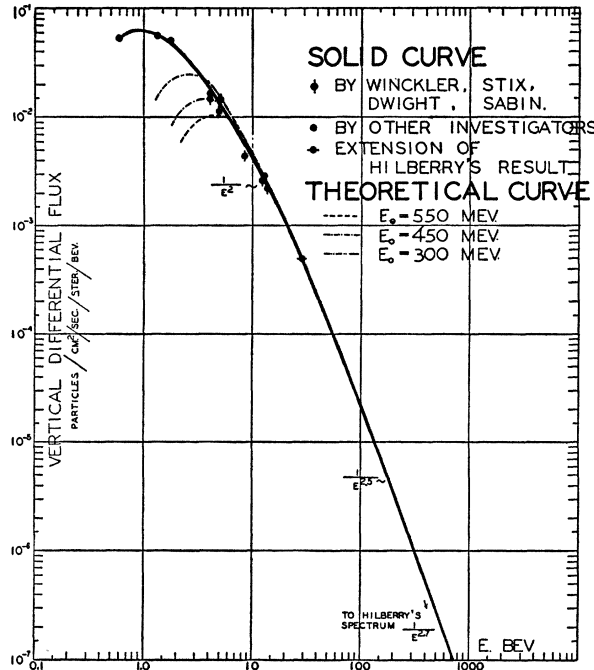


FIG. 1. Differential energy spectrum.

sort of explosions occurred in the nucleus many ages ago. It is hard, at present, to tell which possibility is closer to the truth, but if we assume that primarily the heavier elements are ejected from stars and that protons are the break-up products of these elements by collisions with the interstellar gases, then the first mechanism would give the correct relative abundances of the different components in cosmic radiation and the day-night difference of the intensities of the heavier nuclear components would follow as a direct consequence.⁹ Detailed discussions will be given in Secs. IV and V.

The following data concerning our galaxy and solar system are used for calculations and for comparison:

- (1) Radii of our galaxy: $R_1 = 4.63 \times 10^{22}$ cm, $R_2 = R_1/5$.
- (2) Average distance between stars: 1.7×10^{17} cm.
- (3) Average density of the interstellar matter: 10^{-24} g/cm³.
- (4) Solar system: Near meridian plane of the galaxy and 3.09×10^{22} cm from the center.

II. THE MECHANISM OF ACCELERATION OF THE PROTON COMPONENT

The mechanism of acceleration of the proton component is described in Fermi's paper.³ In order to use it as a basis for further deductions, we reproduce it here with some slight alterations.

A charged particle gains energy by "collisions" with wandering magnetic clouds in interstellar space. The average energy gain per "collision" is given by:

$$\delta w = B^2 w, \quad (1)$$

⁹ J. J. Lord and M. Schein, Phys. Rev. **80**, 304 (1950).

where w is the energy (including the rest energy) of the particle before "collision," and B is the average velocity of the magnetic clouds in units of the velocity of light. The value of B is taken to be of the order of 10^{-4} in accordance with the value given in Fermi's paper. If ϵ is the energy loss in passing through the interstellar matter between collisions, then the energy of the particle after N collisions is given by:

$$w(N) = (1 + B^2)[w(N-1) - \epsilon], \quad (2)$$

if it is not eliminated by nuclear collisions with interstellar gases.

Let ρ be the average density of the interstellar matter, D be the average length of path traversed by a proton between collisions, and q be the energy loss of a proton per g/cm² of material traversed. Then $\epsilon = q\rho D$.

The value of q is tabulated in Fermi's paper and can also be deduced from Smith's results.¹⁰ For high energy protons it is easy to show that the energy dependence of q can be represented adequately by the following form:

$$q = a + b/(w - ah)^x, \quad (3)$$

where $h = D\rho/B^2$ and a , b , and x are three adjustable constants. This form enables us to integrate Eq. (2) exactly. Agreement with the results of Fermi and Smith is achieved by suitably altering the three constants a , b , and x , after we determine the value of h in the next section.

On inserting the value of ϵ in Eq. (2) and then performing the integration, we get:

$$\{[w(N) - ah]^{x+1} - bh\} = \{[w(0) - ah]^{x+1} - bh\} \exp[(x+1)B^2N], \quad (4)$$

if we neglect unity as compared with $1/B^2$.

It is clear from Eq. (4) that a particle will undergo an acceleration or a deceleration according to whether $w(0)$ is greater or less than $[(bh)^{1/(x+1)} + ah]$. Thus, $[(bh)^{1/(x+1)} + ah]$, is the minimum initial energy for subsequent acceleration, and this value minus the rest energy of the proton is called the injection energy of the proton in Fermi's paper

If we put

$$y = \left\{ \frac{[w(N) - ah]^{x+1} - bh}{[w(0) - ah]^{x+1} - bh} \right\}^{1/(x+1)}, \quad (5)$$

then Eq. (4) becomes simply

$$y = \exp(B^2N). \quad (6)$$

III. THE SPECTRUM OF THE PROTON COMPONENT

Recently, Winckler, Stix, Dwight, and Sabin¹ analyzed the spectrum of the proton component. They combined their measurements in the range of $E \approx 4$ Bev to $E \approx 14$ Bev with Hilberry's spectrum¹¹ in the high energy region and with other measurements in the low energy region in order to produce a smooth curve over the entire range. Modifying their curve slightly at the junction of their results and the extension of Hilberry's

¹⁰ J. H. Smith, Phys. Rev. **71**, 32 (1947).

¹¹ N. Hilberry, Phys. Rev. **60**, 1 (1941).

spectrum to make a smoother connection, and converting the integral energy spectrum to a differential energy spectrum, we have the solid curve given in Fig. 1. The slope of each section of the curve is shown in the same figure. Using Fermi's acceleration mechanism as basis, we now investigate the nature of the proton source which will produce this spectrum.

Let Sdt protons of $w(0)$ greater than the minimum initial energy be injected at some point at a distance r from the earth in a time interval dt at $t=0$. Some of them will reach the earth after many collisions with the wandering magnetic clouds. If r is much larger than the average linear distance, L , traversed between collisions, so that the principle of random flight is valid, then the probability density of finding those protons at time t on the earth is

$$P_0(r, t)dt = [Sdt/(2\pi\xi^2)^{1/2}] \exp(-r^2/2\xi^2)$$

provided the loss of protons caused by nuclear collisions with interstellar gases is neglected. Here $\xi^2 = 2NL^2/3$, and N is the total number of collisions in $(0, t)$.

The absorption cross section of a fast proton per nucleon due to nuclear collision is $\sigma_p \approx 2.5 \times 10^{-26}$ cm². Let $\lambda = M/\sigma_p \rho$ be the absorption mean free path and $T = \lambda/\bar{v}$ be the absorption lifetime, where M is the mass of proton, ρ is the average density of the interstellar matter, and $\bar{v}$ is the average velocity of the proton. Then the probability of finding a proton of life greater than t is $\exp(-t/T)$. Taking this factor into account, the probability density of finding the protons which originated from a distance r at $t=0$ on the earth is then equal to

$$P(r, t)dt = \frac{dt}{(2\pi\xi^2)^{1/2}} \exp(-r^2/2\xi^2) \exp(-t/T) \quad (7)$$

for $S=1$.

Putting $s = \sigma_p h/M$ and $R^2 = 4L^2/3B^2$ for simplicity, we have from Eq. (6) and the relation $\bar{v}t = ND$,

$$P(r, y)dy = \frac{sT}{(\pi R^2 \ln y)^{1/2}} \exp(-r^2/R^2 \ln y) \frac{dy}{y^{1+s}} \quad (8)$$

The spectrum of the primary proton component depends, therefore, on the position of the source from which protons originated.

(A)

Case 1. The source is uniformly distributed throughout the whole galaxy and ejects gases at a constant rate.

The spectrum of the proton component observed on the earth for a source ejecting gases at a constant rate is given roughly by:

$$f(E)dE = \left[\int_{E_i}^E \int_V S(E_0)P(r, y)(dy/dE)dE_0 d\tau \right] dE, \quad (9)$$

where $S(E_0)dE_0d\tau$ is the number of protons of kinetic energy between E_0 and E_0+dE_0 ejected from $d\tau$ per unit time, E_i is the minimum initial kinetic energy, and V is the volume of the galaxy. If the distribution of the

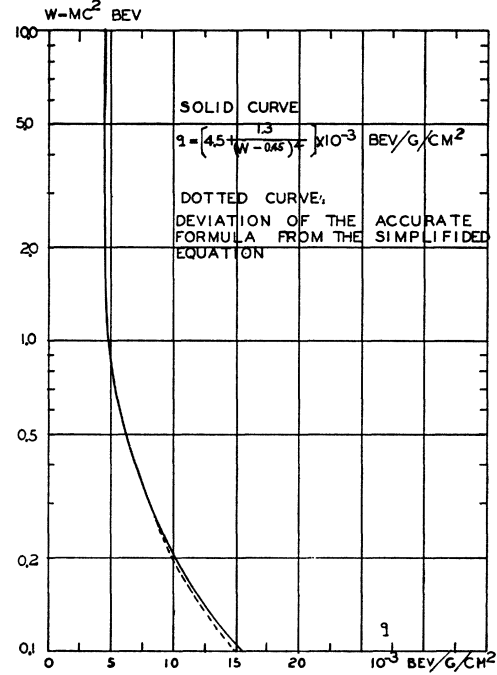


FIG. 2. Plot of q from Eq. (11).

sources is uniform throughout the whole galaxy, as is the case, for instance, with the randomly distributed supernovae, the value of Eq. (9) is then approximately:

$$f(E)dE = (Ts dE/E^{1+s}) \int_{E_i}^E E_0^s S(E_0) dE_0, \quad (10)$$

when we put $y = E/E_0$ (see next Section) and neglect the small difference in the high energy region due to the finiteness of the dimension of the galaxy.

As far as present knowledge goes, the mechanism of ejection of high energy particles from stars (being similar to the explosions in supernovae, according to the most recent speculations) gives, at maximum, energies up to only a few BeV per nucleon. For $E \geq$ the maximum initial kinetic energy per nucleon, Eq. (10) becomes simply

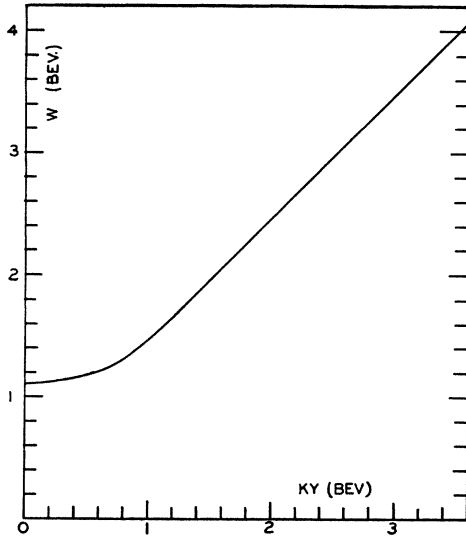
$$f(E)dE = (K/E^{1+s})dE,$$

where K is a constant. This is the well-known inverse power law, which is a good approximation only in the high energy region.

It must be noticed that a uniform source, even if it is very weak, yet may make an important contribution to the low energy region. This can be seen from the rapid increase of $f(E)$ with decreasing E in Eq. (10).

(B)

Case 2. "Point source." Equation (8) gives the spectrum of the proton component if the protons originate from a "point source" emitting at a constant rate with a definite initial energy. It is clear that Eq. (8) consists of two factors: the first contains $(\ln y)$ as an argument and is important for small values of y , while the other is

FIG. 3. Plot of $W(N)$ vs y , for $k=1$.

important for large values of y . $P(r, y)$ increases rapidly from zero to a maximum and then decreases. It varies as $1/y^{1+s}$ when $y = \exp(2r^2/3R^2)$ and becomes gradually steeper than $1/y^{1+s}$ with increasing y . This feature, together with the shape of the experimental energy spectrum, suggests $s=1.5$ in Eq. (8). From this value of s ($=\sigma_p h/M$), the following results can be obtained:

(a) $h = D\rho/B^2 = 100 \text{ g/cm}^2$; $D \approx 10^{18} \text{ cm}$.

(b) By adjusting the constants a , b , and x in Eq. (3) to fit Fermi's and Smith's results, the energy loss of a high energy proton per g/cm^2 of material traversed in the interstellar space can be represented by:

$$q = [4.5 + 1.3/(w - 0.45)^4] \times 10^{-3} \text{ BeV/g/cm}^2. \quad (11)$$

The comparison of Eq. (11) and the exact results is shown in Fig. 2.

(c) The minimum value of $w(0)$, denoted by $w(0)_i$, is given by $[(w(0)_i - 0.45)^5 - 0.31] = 0$ which gives $w(0)_i = 1.12 \text{ BeV}$. The injection energy is, therefore, about 0.2 BeV.

(d) The value of y is then

$$y = \left[\frac{(w(N) - 0.45)^5 - 0.13}{(w(0) - 0.45)^5 - 0.13} \right]^{1/5}. \quad (12)$$

For large $w(N)$,

$$y \approx k(w(N) - 0.45) \approx kE,$$

where $k = 1/[(w(0) - 0.45)^5 - 0.31]^{1/5}$. If $w(0)$ is also large, y is simply equal to E/E_0 , where E_0 is the initial kinetic energy. The dependence of $w(N)$ on y for $k=1$ is given in Fig. 3.

The adjustment of Eq. (8) to fit with the experimental curve is facilitated by the fact that when we plot Eq. (8) on log-log paper the shape of the curve is not altered if we assume a different initial energy or multiply by an arbitrary constant. Therefore, taking k to be unity and fitting our curve with two arbitrary points of the experi-

mental result (we take $E=7.4 \text{ BeV}$ and 20 BeV), we get:

$$P_1(r, y) = [115/(\ln y)^3] e^{-7.4/\ln y} (1/y^{2.5}). \quad (13)$$

The comparison with the experimental results is shown in Fig. 1, where the theoretical curves of different initial energies are obtained by mere linear translations. A change of the parameter $w(0)$ causes a horizontal translation, and an arbitrary multiplier is introduced to effect a vertical linear translation, which brings the high energy portion of the curves to coincidence except for a small discrepancy which is not indicated.

The above results indicate that a "point source" seems to be a close guess. If this is not merely a coincidence, then the assumed source is probably the nucleus of our galaxy because it is the only place, as far as we know, which can supply the necessary energy. Assume that $D \approx L$ and $\rho \approx 10^{-24} \text{ g/cm}^3$, we find from the value of h and of r^2/R^2 that the distance of our assumed source is just the distance of the center of our galaxy from the earth, $3.1 \times 10^{22} \text{ cm}$.

(C)

Discussion—the low energy end of the spectrum. Before we give a detailed discussion of Case 2, we should emphasize one point which is, perhaps, the weakest part of the whole picture presented in this paper.

Our speculations have been based on the experimental results of Winckler *et al.*,¹ and we have indicated that the most probable spatial distribution of proton sources is that which would most closely produce these results. But we have ignored the possibility that the observed spectrum may be distorted. Thus, the following facts, for instance, have not been considered: (1) the spectrum may be distorted by the magnetic field of the sun; (2) the observed cosmic-ray protons are presumably mixed up with the protons produced through nuclear collisions by cosmic-ray protons which we have considered as lost in our calculations. The dipole magnetic field of the sun probably affects only low energy particles,¹² but the secondary protons will distort the spectrum even up to the high energies. The purpose of the following discussions is to show that the distortion seems unlikely to be large except at the low energy end.

The protons produced through nuclear collisions by cosmic-ray protons tend to distribute themselves uniformly throughout the whole galaxy. These protons will be again accelerated according to Fermi's mechanism. Thus, we may consider these secondary protons as a uniformly distributed source, and, therefore, Eq. (10) is applicable for this case except that E does not have a definite maximum.

Differentiating Eq. (10) with respect to E , we have:

$$S(E) = (1/Ts)[(1+s)f(E) + Ef'(E)].$$

Let us put

$$f(E) = kK/E^s.$$

¹² The dipole magnetic field of the sun was proposed in 1939 as an explanation of the cut-off of the energy spectrum below 4 BeV. Now it is believed that the magnetic field is perhaps much weaker than thought before.

Here K is a constant, and k and x are *slowly-varying* functions of E ; x varies from some negative value to $1+s$, and k varies from 0 to 1. Then

$$S(E) \approx (1+s-x)f(E)/Ts.$$

The last equation shows that if the secondary protons are principal sources, $S(E)$ would have a maximum at $E \approx 1$ Bev as $f(E)$. This seems unlikely to be the possible energy spectrum of the protons produced by cosmic-ray protons through nuclear collisions. Arguing on this basis, the turning-off of the primary energy spectrum indicates perhaps that $S(E)$ is rather small. It is, therefore, considered together with supernovae as a source which plays an important part only in the low energy region and will not be mentioned in the next section.

IV. GENERAL DISCUSSION OF THE SOURCE

The nucleus of our galaxy is presumably a cluster of stars. The density of stars in our galaxy might be put in the following form:

$$D(a) = (D_0/a) \exp(-a^2/a_0^2), \quad (14)$$

where a is the distance measured from the center of our galaxy, and D_0 and a_0 are two constants (a simplified form of Oort's description of our galaxy). If stars are able to eject high energy particles (up to 1 Bev per nucleon) as proposed by different authors,^{7,8} then our result would mean that the cosmic radiation originates from stars and is then accelerated according to Fermi's mechanism in interstellar space. In fact, if we integrate Eq. (8) over the whole space with the weight function (14), we obtain a curve similar to that of Fig. 1 except that the maximum of the spectrum shifts toward lower energies. This is to be expected since the distribution (14) is highly concentrated. When we choose $E_0 = 1$ Bev and $a_0 = r^2/7.4$ for simplicity (r is the distance from the earth to the nucleus), the maximum occurs at $E \approx 2$ Bev.

On the other hand, if stars are unable to eject high energy particles, our result would mean that something corresponding to a series of explosions occurred in the nucleus of the galaxy over an extended period of time many ages ago. Our galaxy belongs to the late stage of the normal spiral type. A normal spiral type has its arms fully developed before condensation occurs. This could be understood if the latter hypothetical mechanism were correct, because the ejection of a large amount of gas would result in the reduction of the mass of the nucleus, causing the arms to be developed by centrifugal force.

At the present time it is hard to tell which of the hypothesis discussed above is the more nearly correct, but the author favors the first one, because it would solve the problem of the heavier nuclear components of cosmic radiation, as will be shown in the next section.

V. THE HEAVIER NUCLEAR COMPONENTS OF COSMIC RADIATION

(A) The Relative Abundances

In dealing with the heavier nuclear components we encounter one difficulty; namely, the high minimum

initial energies required for subsequent acceleration. Let $w(N)$ now denote the energy per nucleon of a nucleus A just after N collisions. Integration of Eq. (2), with ϵ as a constant, yields:

$$[Aw(N) - \epsilon/B^2] = [Aw(0) - \epsilon/B^2] \exp(B^2 N). \quad (15)$$

ϵ/B^2 (denoted by $Aw(0)_i$), is then the required minimum energy of the nucleus under consideration. If Z is the nuclear charge, the minimum initial kinetic energy per nucleon¹³ (denoted by E_i), is about $0.2Z^2/A$.

Now let us assume that mostly heavier nuclei of charge $Z \geq 6$ are originally ejected from the source and that protons and α -particles are the break-up products of such nuclei, resulting from collisions with the interstellar gases.¹⁴ If the source is far from the earth and protons and α -particles exist in equilibrium with the heavier nuclei, then two cases must be distinguished:

$$(1) w(0) \geq w(0)_i$$

Let us start originally with 6 nuclei of C, N, and O and 2 heavier nuclei of average atomic weight, $A = 30$. The ratio 6:2 is that existing among these gases in the interstellar medium. Assume that the break-up fragments of a nucleus A consist of $\frac{1}{6}A$ α -particles and $\frac{1}{3}A$ protons and neutrons (which will decay to be protons).¹⁵ Take the absorption cross section of He as

$$\sigma_{\text{He}} = (4)^{\frac{1}{2}} \sigma_p = 6.3 \times 10^{-26} \text{ cm}^2,$$

and the cross sections of the C, N, O group as¹⁶

$$\sigma_{\text{C, N, O}} = 20 \times 10^{-26} \text{ cm}^2,$$

and that of $A \geq 30$ as¹⁶

$$\sigma_M = 30 \times 10^{-26} \text{ cm}^2.$$

Then the number of α -particles (N_{He}) when in equilibrium with nuclei of charge $Z \geq 6$ is

$$N_{\text{He}} = \frac{14}{6} \frac{\sigma_{\text{C, N, O}}}{\sigma_{\text{He}}} N_{\text{C, N, O}} + \frac{30}{6} \frac{\sigma_M}{\sigma_{\text{He}}} N_M \approx 90,$$

and the number of fast protons (N_p) is

$$N_p = \frac{14}{3} \frac{\sigma_{\text{C, N, O}}}{\sigma_p} N_{\text{C, N, O}} + \frac{30}{3} \frac{\sigma_M}{\sigma_p} N_M + 4 \frac{\sigma_{\text{He}}}{\sigma_p} N_{\text{He}} \approx 1400.$$

Thus, on the assumption that only nuclei of charge $Z \geq 6$ are ejected, we predict that the ratio of elements in cosmic radiation with energies above a certain minimum energy per nucleon should be

$$\text{H:He:(C, N, O):(Z} \geq 10) = 1400:90:6:2.$$

This is of the order of magnitude of the observed ratio, as first pointed out by Bradt and Peters.¹⁶

¹³ Gases ejected from stars are presumably only partially ionized. The stripping cross section per electron by collision is about 10^{-20} cm^2 , so that we can assume that all of the nuclei are bare except those coming directly from the sun.

¹⁴ The absorption cross sections of the heavier elements are sufficiently large so that no serious errors are introduced if we assume that the protons originate directly from a star in calculating the spectrum of the proton component.

¹⁵ J. B. Harding, Phil. Mag. 11, 530 (1949).

¹⁶ H. L. Bradt and B. Peters, Phys. Rev. 77, 54 (1950).

$$(2) w(0)_i \geq w(0) \geq 1.12 \text{ Bev}$$

In this case the heavier nuclei under consideration are unable to be accelerated, but the break-up products, protons and α -particles, have their energies still above the minimum required energy 1.12 Bev per nucleon. Because the source is far away from the earth, heavier nuclei will be slowed down and absorbed before they reach the earth.

Thus, a distant source, as the nucleus of our galaxy would be, for instance, gives too low abundances for heavier nuclear components. This difficulty can be overcome if we adopt the first mechanism discussed in the last section; that is, if we assume that stars are the original source of cosmic radiation.

The closer stars give much higher relative abundances for heavier nuclei for the following two reasons: (1) the equilibrium condition has not been reached, and (2) low energy nuclei still have a chance to reach the earth. Specifically, the sun would give an important contribution to the low energy heavier nuclear components, and, therefore, the day-night difference of intensities of these particles⁹ would follow as a direct consequence.

(B) The Energy Spectra of The Heavier Nuclear Components

The energy spectrum of the α -particles can be obtained by the same argument as that by which we obtained the spectrum of the proton component. It is practically:

$$P(r, y)dy = \frac{KS(E_0)}{(\ln y)^{\frac{1}{2}}} e^{-7.4/\ln y} \frac{dy}{y^{4.8}} \quad (16)$$

for a given initial energy, where y stands for $[4w(N) - \epsilon/B^2]/[4w(0) - \epsilon/B^2]$, K is a constant and $S(E_0)$ is the number of α -particles ejected per sec with initial kinetic energy E_0 , ($\approx (4w(0) - \epsilon/B^2)$).

Equation (16) has a maximum at $y \approx 3$. The value of the maximum is $K'S(E_0)$ where K' is a constant. Because the spectrum for a given initial energy is so steep that the shape of the *observed* spectrum would be then roughly given by:

$$f(E)dE = (dE/E)K'S(E/3)$$

when we write $y \approx E/E_0$. This resembles more or less the initial energy spectrum except that the observed spectrum may have a steep "tail" in the high energy region.

From considerations similar to the above discussion, the energy spectra of the heavier nuclear components would all resemble the initial energy spectra.¹⁷

It is interesting to notice that by a comparison of Eq. (8) with Eq. (13) we are able to estimate the total number of nucleons of kinetic energy ≥ 0.2 Bev ejected from each stars if our hypothetical mechanism is correct. Let S be the total number of nucleons of $w(0) \geq w(0)_i$

ejected per second. Then

$$SsT\bar{v}/3(\pi R^2)^{\frac{1}{2}} \approx 115\pi.$$

This gives $S \approx 10^{44}$ particles per second. There are 10^{11} stars in our galaxy. The number of nucleons of kinetic energy ≥ 0.2 Bev ejected from each stars is then 10^{33} per second. This is about 10^{-24} of the mass of the sun, and the energy is about 10^{-4} of the total electromagnetic energy emitted per second from the sun.

VI. THE EFFECT OF THE "REFLECTIVITY" OF THE GALACTIC BOUNDARY

In this section we shall estimate the effect of the boundary condition of our galaxy. The question is: How much does the trapping magnetic field of our galaxy, if it exists at all, affect the spectra of cosmic radiation? If there is such a magnetic field and if it is strong, we can consider the boundary of the galaxy to be a perfect reflector. On the other hand, if there is no magnetic field to trap cosmic radiation, we can consider the boundary to be a perfect absorber. The theory of random flight with a reflecting boundary and with an absorbing boundary provides an easy way of estimating this effect.

In order to give only the qualitative features for the two cases, we simplify the condition as follows: We suppose the galaxy to be a sphere of radius a and assume that cosmic radiation originates only from the center of the galaxy. For a given initial energy, the spectrum of the cosmic radiation (say the proton component), is given by:

$$P(r, y)dy = \frac{T_s}{(\pi R^2 \ln y)^{\frac{1}{2}}} \times \exp\left(-\frac{r^2}{R^2 \ln y}\right) \left\{ 1 \pm \exp\left[-\frac{4a(a-r)}{R^2 \ln y}\right] \right\} \frac{dy}{y^{2.5}},$$

where the notation is the same as in Eq. (8). The plus sign is for a perfectly reflecting boundary, and the minus sign is for a perfectly absorbing boundary. For a perfect reflector, the spectrum will be flatter in the high energy region, and for a perfect absorber, it will be steeper. But the effect is so small that it could not be shown in Fig. 1.

VI. THE TIME SCALE OF THE GALAXY

If we assume that the state of interstellar space has not changed since the "birth" of the galaxy and that the protons of the highest energy in the cosmic radiation have been accelerated ever since, then Fermi's theory serves as a new method for estimating the age of our galaxy. The highest proton energy observed is about 10^8 Bev. Assuming that such a particle had an initial kinetic energy of 2 Bev, then the life of this particle is 6×10^{16} sec, which is about two billion years, from which we conclude that the age of the galaxy is greater than two billion years.

¹⁷ M. S. Vallarta, Phys. Rev. 77, 419 (1950).